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SIMULATION OF EARTH PLANETARY ORBITS USING A MODIFIED INVERSE SQUARE MODEL

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Abstract: This paper is aimed at simulating earth planetary orbits using a Modified Inverse Square Model. The environmental and climatic challenges as currently witnessed appear to suggest that the usual Inverse Square law of earth planetary motion may be far from being correct. The modified Inverse Square Model was simulated using the fourth order Runge-Kutta method implemented through the ode45 solver of the Mat lab. The parameter β in the Model acts in switching on the modification of the Inverse square Law. When β =0, there is no modification of the model and the usual earth elliptic planetary orbit was ensured. With B > 0, the orbits of the earth showed interesting patterns (depending on the strength of β). The earth at certain period of the year moves very close to the sun and very remote from the sun at some other times. The results obtained shows that the intensity of the sun is very high which induces high temperature, rapid vaporization to the earth which causes sea level to rise and change in the amount of and pattern of precipitation or rainfall and expansion of subtropical deserts, retreat of glaciers etc; it also causes changes in the frequency and intensity of extreme weather events, and changes in agricultural yields or suppression of vegetation. It is suggested that illegal bush burning and deforestation should be avoided to decrease Global warming (GW) and depletion of Ozone layer such that the earth will not be close to the sun but remote from the sun for the comfortability of earth satellites.

Keywords: Planetary motion, Inverse square law, Modified inverse square model and Phase space

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Introduction

The study of planetary motion and mathematical modelling of orbits began with first attempt to predict orbital motion in space (Adams and Rider, 1987), although in ancient time the causes of the motion remained a mystery. One of the most prominent features of the universe is motion (Alfriend and Yan, 2005). All forms of matter (solid, liquid and gases) are composed of atoms which are in rapid motions about their equilibrium positions. Even galaxies have motions with respect to other galaxies, all stars have motion, and the planets have motions against the background of the stars (Ballard, 1980). Motion is a phenomenon we must learnt to deal with in all levels, if we are to understand the world round us. In fact; events what captured our attention most quickly in everyday life are those that involve motion. Our technology is highly dependent on our knowledge of motion (Barker and Stoen, 2001).There is thus every reason for one to go into a detail study of motion. Motion involves a change of position and state of a body, depending on time. The position or state of a body cannot be changed in the absence of force, therefore, force is a cause of motion. Forces may not always cause motion, however. For example, gravitational force acts on a body as it perform work, yet it remains stationary. That means if an object moves with uniform velocity, no force is required for the motion to be maintained.

Planetary motion is of special significance because it played an important role in the conceptual history of the mechanical view of the universe (Beste, 1978). Few theories have affected western civilization tremendously as much as Newton's law of motion and the law of gravitation, which together relate the motion of the heavens to the motion of terrestrial bodies (Carter, 1990).

Planetary motion which is much concerned in the study of orbit in central potential refers to the revolution of planets in their orbits around the sun (Chen *et al.*, 2000). Planetary motion is of special importance to life on earth. The orbit of the earth around the sun and the axial rotation are undoubtedly one of the most significant motions in terms of the effects it produces. The rotation causes seasonality while the axial rotation causes day and night (Clohessy and Wiltshire, 1960). The climatic uncertainties over the years have largely been attributed to human activities such as gas sharing; depleting the ozone and the global warming (Clarke, 1945). Classical mechanics has used the inverse square law to predict the period of the earth motion which is used in anticipating one form of weather or the other (French, 1971).The fluctuations of weather patterns suggest the need to revisit the inverse square. This research seeks to prove if there are inadequacies in the use of inverse square law to describe planetary motion. The revolution of the earth causes the different constellations to come to view.

Another spectacular phenomenon associated with planetary motion, (specifically motion of the earth-moon system) is the eclipse, this occurs whenever the shadow of either the moon or the earth falls upon the other (Greene, 1997).

Observational astronomy had yielded data of marvellous accuracy for years, few theories on celestial mechanics and prominent laws were involved, which includes Newton's law of universal gravitation and Kepler's law of planetary motion. These laws make the study of planetary motion very possible (Gould and Harvey, 1988).

The one body problem describes the motion of two bodies interacting gravitationally with each other with the given parameters: Masses and the initial velocity coordinates through a central force (Gim and Alfriend, 2003). Computer simulation is going to be used for the one body problem special case (Gurfil and Kholshevnikov, 2006). The result is the system of differential equations describing the body's movement. These equations will be solved numerically, therefore the next sections of the study briefly covers integration method theory. It describes how these methods are used to interpolate the value of the integral by differentiation and how various methods differ from each other. The methods that are used in the computer simulation are: the Euler's method and Runge-Kutta method.

Theoretical Consideration

The Inverse Square Model

Kepler's laws talks about how planets moves but no indication to the direction. It was Newton's analysis of the orbit that also formed part the basis for his law of universal gravitation. He reasoned that the moon was held in orbit by some kind of attraction between the moon and the earth and set about finding a relation between the magnitude of this force and the separation of the bodies. When he was attempting to find a law of force that would be consistent with Kepler's third law, proposed his law of universal gravitation (Halliday, *et al.*, 1987).

Newton's law of universal gravitation states that every particle of matter in the universe attracts every other particle with masses M and m with a force given

$$F_g = -\frac{GMm}{r^2}\hat{r} = -\frac{GMm}{r^3}r \qquad (1)$$

Where F_g is the magnitude of the gravitational force on either particles M and m are masses, the vector $\hat{r}$ is directed from M to m are the negative sign implies that the gravitational force is attractive, that is, it tends to decrease the separation r. G is a fundamental physical constant called the gravitational constant. $G = 6.67 \times 10^{-11}\text{N}.m^2/Kg^2$.

The gravitational force has two characteristics: its magnitude depends only on the separation of the particles, its direction is along the line joining the particles; such a force is called a central force.

The assumption of a central force implies that the orbit of the earth is restricted to $(x - y)$ plane and the angular momentum L is conserved and lies in the third z-direction (Hablani, 1989).

We write

$$\text{L} = (\text{r} \times \text{m} \times \text{v})_z = \text{m}(\text{xvy} - \text{yvx}) \qquad (2)$$

Where we have used the cross product definition L = r × p and p = mv. An additional constraint on the total energy, E is conserved and is given by

$$E = \frac{1}{2}mv^2 - \frac{GMm}{r} \qquad (3)$$

We fixed the coordinate system at the mass M, the equation of motion of the particle of mass m is

$$m\frac{d^2r}{dt^2} = -\frac{GMm}{r^3} \qquad (4)$$

The motion of an object of mass m which moves under the influence of a central force F provides useful relation for writing the equation of motion in component form which is suitable for numerical solutions. It is convenient to represent the force in Cartesian coordinate.

$$F_x = -\frac{GMm}{r^2}\cos\theta = -\frac{GMm}{r3}x \qquad (5)$$

$$F_y = -\frac{GMm}{r^2}\sin\theta = -\frac{GMm}{r3}y \qquad (6)$$

Hence, the equations of motion in Cartesian coordinates for inverse square model are:

$$\frac{d^2x}{dt^2} = -\frac{GM}{r^2}x \qquad (7)$$

$$\frac{d^2y}{dt^2} = -\frac{GM}{r3}y \qquad (8)$$

where $r^2 = x^2 + y^2$ · Equations (7) and (8) are example of coupled differential equations because each equation contains both x *and* y.

The Modified Inverse-Square Model

Consider a satellite with a mass M_{sat} orbiting a central body called the sun with a mass M_c. If the satellite moves in circular motion, then the net centripetal force acting upon this orbiting satellite is given by the relationship

$$F_{net} = \frac{M_{sat}V^2}{r} \qquad (9)$$

This net centripetal force is as a result of the gravitational force which attracts the satellite towards the central body and can be regarded as the Modified Inverse Square Model represented by

$$F = F^0 + F' \qquad (10)$$

Where $F^0 = -\frac{GM_{sat}M_c}{r^2}$ is the unperturbed Inverse Square Law and $F' = -\beta\frac{GM_{sat}M_c}{r^3}$ is the perturbed Inverse Square Law. Then the equation (10) becomes

$$F = -\frac{GM_{sat}M_c}{r^2} - \beta\frac{GM_{sat}M_c}{r^3} \qquad (11)$$

Since equations 9 and 11 are equal, thus

$$\frac{M_{sat}V^2}{r} = -\frac{GM_{sat}M_c}{r^2} - \beta\frac{GM_{sat}M_c}{r^3}$$

Therefore,

$$V^2 = \frac{GM_c}{r} + \frac{\beta M_c}{r^2} \qquad (12)$$

Taking the square root of each side, we have

$$V = \sqrt{\frac{GM_c}{r} + \frac{\beta M_c}{r^2}} \qquad (13)$$

Fixing the coordinate system at the mass, M_c, the equation of motion of the particle of mass M_{sat} is given by

$$M_c\frac{d^2r}{dt^2} = -\frac{GM_{sat}M_c}{r^3} - \beta\frac{M_{sat}M_c}{r^4} \qquad (14)$$

The motion of an object of mass M_{sat} which moves under the influence of a central force F provides useful relation for writing the equation of motion in component form which is suitable for numerical solutions. It is convenient to represent the force in Cartesian coordinate.

$$F_x = -\frac{GM_{sat}M_c}{r^3}\cos\theta - \beta\frac{M_{sat}M_c}{r^4}\cos\theta$$

$$F_x = -\frac{GM_{sat}M_c}{r^3}x - \beta\frac{M_{sat}M_c}{r^4}x \qquad (15)$$

$$F_y = -\frac{GM_{sat}M_c}{r^3}\sin\theta - \beta\frac{M_{sat}M_c}{r^4}\sin\theta$$

$$F_y = -\frac{GM_{sat}M_c}{r^3}y - \beta\frac{M_{sat}M_c}{r^4}y \qquad (16)$$

Hence, the equations of motion in Cartesian coordinates are

$$\frac{d^2x}{dt^2} = -\frac{GM_{sat}}{r^3}x - \beta\frac{M_{sat}}{r^4}x \qquad (17)$$

$$\frac{d^2y}{dt^2} = -\frac{GM_{sat}}{r^3}y - \beta\frac{M_{sat}}{r^4}y \qquad (18)$$

Where $r^2 = x^2 + y^2$. Equations 17 and 18 are example of coupled differential equations because each equation contains both x and y.

Numerical solution of the models

The equations of motion in Cartesian form used in the simulation of planetary orbits are:

$$\frac{d^2x}{dt^2} = -\frac{GM}{r^3}x = f(x,y) \quad (19)$$

$$\frac{d^2y}{dt^2} = -\frac{GM}{r^3}y = f(x,y) \quad (20)$$

The two equations are coupled and in standard form are:

$$X'' = f(x,y,t)$$

$$Y'' = g(x,y,t) \quad (22)$$

These are solved numerically using Mat Lab with ODE 45 module. The algorithm for the module is based on Runge – kutta 4th and 5th scheme as presented in equations (19) and (20). The system of equation which is first transformed into a system of two first order differential equations are as follows:

Considering the inverse square model(equations (7) and (8)),

We set $x = x_1$ and

$$\frac{dx}{dt} = \frac{dx_1}{dt} = x_2$$

$$\frac{d^2x}{dt^2} = \frac{d^2x_1}{dt^2} \Rightarrow \frac{dx_2}{dt} = -\frac{GM}{r^2}x_1$$

$$\frac{dx_2}{dt} = -\frac{GM}{r^3}x_1$$

Again,

Let $y = x_3$

$$\frac{dy}{dt} = \frac{dx_3}{dt} = x_4$$

$$\frac{d^2y}{dt^2} = \frac{d^2x_3}{dt^2} \Rightarrow \frac{dx_4}{dt} = -\frac{GM}{r^3}x_3$$

$$\frac{dx_4}{dt} = -\frac{GM}{r^3}x_3$$

The new system of first order equations is given as below

$$\frac{dx_1}{dt} = x_2 \quad (23)$$

$$\frac{dx_2}{dt} = -\frac{GM}{r^3}x_1 \quad (24)$$

$$\frac{dx_3}{dt} = x_4 \quad (25)$$

$$\frac{dx_4}{dt} = -\frac{GM}{r^3}x_3 \quad (26)$$

Considering the modified inverse square model (equations (17) and (18)),

We set $x = y_1$

$$\frac{dy_1}{dt} = y_2 \quad (21) \quad (27)$$

$$\frac{dy2}{dt} = -\frac{GM_{sat}}{r^3}y_1 - \beta\frac{M_{sat}}{r^4}y_1 \quad (28)$$

Again, let $y = y_3$

$$\frac{dy_3}{dt} = y_4 \quad (29)$$

$$\frac{dy_4}{dt} = -\frac{GM_{sat}}{r^3}y_3 - \beta\frac{M_{sat}}{r^4}y_3 \quad (30)$$

Results

In phase space of the inverse square model, the solution of equations (23), (24), (25) and (26) were obtained and the time used to observed the motion was (0, 11) and the initial conditions are $x_1(0) = 0.65;\ x_2(0) = -0.5;\ x_3(0) = 0.8;$ and $x_4(0) = 1$ with the parameters; G = 1.0, M = 1.0 and r = $\sqrt{x(1)^2 + x(3)^2}$. The set of values for x and y where produced and used to plot a trajectory of the motion as shown in Figure 1.

In phase space of the modified inverse square model, the program for circular orbit was adapted to produce orbits with perturbation effect. The orbits obeyed the force law F = $-\frac{GM_{sat}}{r^2}$ - $\beta\frac{M_{sat}}{r^3}$ instead of F = $-\frac{GM_{sat}}{r^2}$. The same initial conditions and parameters used for the previous simulation were also used to simulate this model but the time span was (0, 45) to give a clear picture of the perturbation effect. A series of values of y and x were generated and used to plot the trajectory of the motion as shown from Fig. 2 to 7 for different values of β.

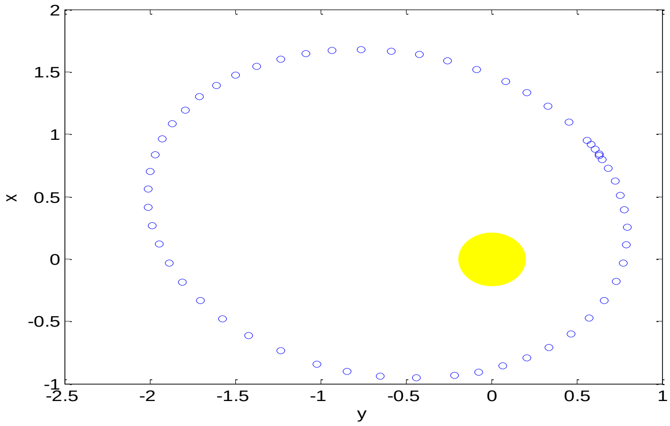

Figure 1. Phase Space of the Inverse Square Law

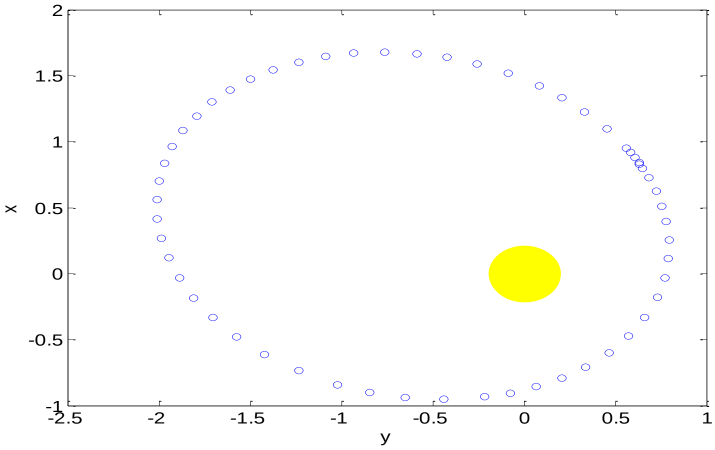

Figure 2. Phase Space of the Modified Inverse Square Law (β = 0)

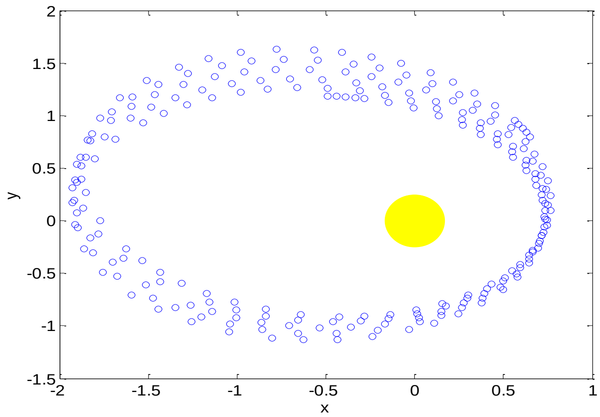

Figure 3. Phase Space of the Modified Inverse Square Model (β = 0.04)

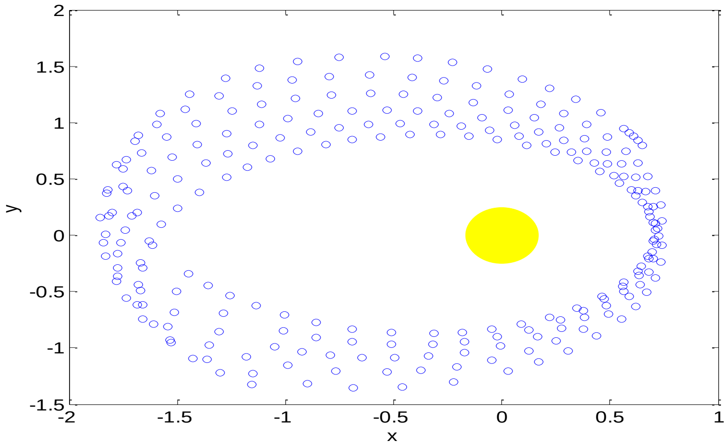

Figure 4. Phase Space of the Modified Inverse Square Model (β = 0.08)

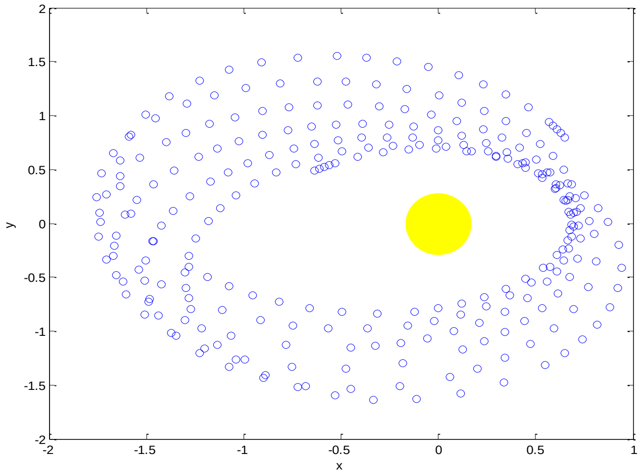

Figure 5. Phase Space of the Modified Inverse Square Model (β = 0.12)

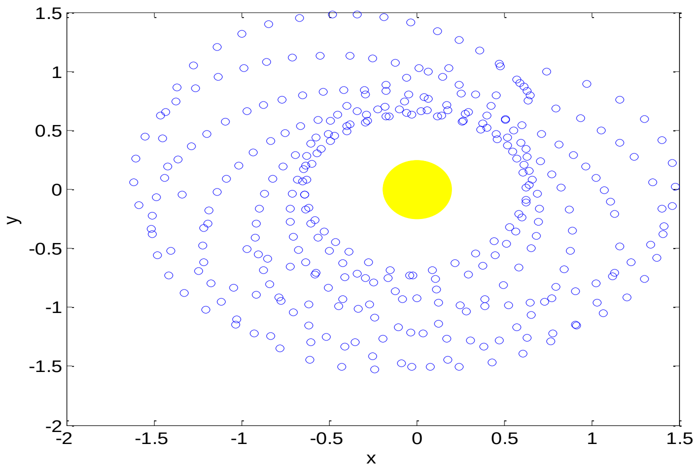

Figure 6. Phase Space of the Modified Inverse Square Model (β = 0.20)

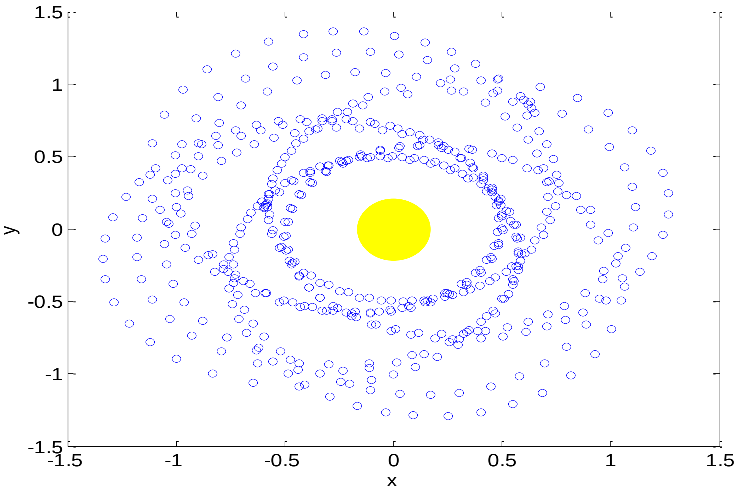

Figure 7. Phase Space of the Modified Inverse Square Model (β=0.39)

Discussion

The simulation of planetary orbits with the condition for circular orbit yields result that is close to the one predicted by analytic method. Analytically the condition for circular orbit is $r^2 = x^2 + y^2$, this is exactly equation of a circle. Figure 1 represents the graph of x versus y from the simulation of the orbit. The graph describes the motion of the earth about the sun. The graph seems to be an ellipse at a glance which shows that the earth orbits the sun in an elliptic manner.

The graph of y versus x in Figure 2 shows that the planet will not describe single elliptical path if subjected to the force law of the form $F = {k}/{r^2} + {\beta}/{r^3}$

where k is a constant, F = force keeping planet in orbit and β is also a constant. When β = 0, the usual elliptic orbit was assured and therefore there was no modification. In Figure 3 when β was 0.04 we observe that the planet retraces itself in different paths. Figure 4 shows that the number of orbits were increased when the value β was increased to 0.08. A critical look at figures 5 and 6 show that, as the value of β increases, the more orbits are generated and also the orbits will no longer be exact eclipse that close on themselves, instead, the orbits retrace themselves. Figure 7 shows that the planet moves towards the sun gradually whenever the value of β is increased after each time it completes an orbit and very remote from the sun at some other times.

Conclusion

The simulation of earth planetary orbits using the inverse Square Law gives the usual earth elliptic orbit as expected. This serves as an indication that the turning on of perturbation gives the correct result and not just error propagation in the algorithm. For β > 0, the earth elliptic orbits not only shifted from the usual orbit but traces different paths about the sun. At certain time, the orbits are very close to the sun and at other times very far from the sun. The fluctuation of the earth's orbits about the sun has significant effects on the earth climate and weather. The fluctuation of the earth's climate and global warming may be correctly linked to the earth fluctuating orbits. The Modified Inverse Square Model (MISM) thus described more closely the earth's climatic changes than the usual Inverse square Model. MISM revealed that the fluctuation of the weather pattern and the irregularity of the earth's orbit is not only as a result of the utilization of energy resources in running factories, bush burning, deforestation and burning of fossil fuels but also the proximity of the earth orbit to the sun and very remote from the sun at some times.

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References

Adams, W.S. and Rider, L. 1987. Circular Polar Constellations Providing Continuous Single or Multiple Coverage above a Specific Latitude. J. Astr. Sci., 35(2): 155–192.

Alfriend, K.T. and Yan, H. 2005. Evaluation and Comparison of Relative Motion Theories. J. Gui. Cont. Dyn., 28 (2): 254–261.

Ballard, A.H. 1980. Rosette Constellations of Earth Satellites. IEEE Trans. Aero. Elec. Sys., 16 (5): 656–673.

Barker, L. and Stoen, J. 2001. Sirius Satellite Design: The Challenges of the Tundra Orbit in Commercial Spacecraft Design. AAS Guidance and Control Conference, Breckenridge, Colorado, AAS. pp: 01-071.

Beste, D.C. 1978. Design of Satellite Constellations for Optimal Continuous Coverage. IEEE Trans. Aero. Elec. Sys., 14(3): 466–473.

Carter, T.E. 1990. New Form for the Optimal Rendezvous Equations NSear a Keplerian Orbit. J. Gui. Cont. Dyn., 13(1): 183–186.

Chen, X., Steyn, W.H., and Hashida, Y. 2000. Ground-Target Tracking Control of Earth- Pointing Satellites. AIAA Guidance, Navigation and Control Conference and Exhibit, Denver, Colorado, AIAA. pp: 00-4547.

Clarke, A.C. 1945. Extra-Terrestrial Relays. Wireless World and Radio Review., 51(10): 305–308.

Clohessy, W.H. and Wiltshire, R.S. 1960. Terminal Guidance System for Satellite Ren- dezvous. J. Astr. Sci., 27(9): 653–678.

French, A.P. 1971. Newtonian Mechanics; An Introductory Level Text with more than a Cursory Treatment of Planetary Motion. W. W. Norton and company. pp: 204-400.

Gim, D.W. and Alfriend, K.T. 2003. State Transition Matrix of Relative Motion for the Perturbed Noncircular Reference Orbit. J. Gui. Cont. Dyn., 26 (6): 956–971.

Gould and Harvey, 1988, Introduction to Level Computer Simulations Application to Physical System. 3rd Edition Pearson Education, inc. Addition Wesley, San Francisco, U.S.A. pp: 79-80.

Greene, B. 1997. Poincare and the Three Body Problem. American mathematical society. Rhode Island, U.S.A. pp: 7.

Gurfil, P. and Kholshevnikov, K.V. 2006. Manifolds and Metrics in the Relative Spacecraft Motion Problem. J. Gui. Cont. Dyn., 29 (4): 1004–1010.

Hablani, H.B. 1989. Design of a Payload Pointing Control System Tracking Moving Objects. J. Gui. Cont. Dyn., 12(3): 365–374.

Halliday, Resnik and Walker, 1987. Fundamentals of Physics.7th edition. John Wiley and Sons U.S.A. pp: 203 & 343.